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PRACTICAL METHODS FOR THE COMPENSATION AND CONTROL OF  
MULTIVARIABLE SYSTE. (U) BROWN UNIV PROVIDENCE RI  
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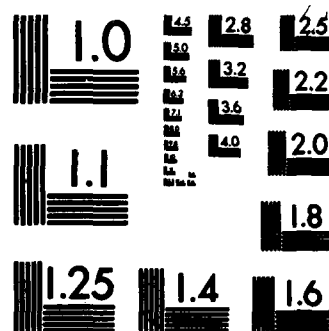
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PRACTICAL METHODS FOR THE COMPENSATION AND  
CONTROL OF MULTIVARIABLE SYSTEMS

AFOSR-82-0034

Research Objectives:

The primary purpose of this research effort has been to develop practical methods for the analysis, compensation and control of multivariable systems such as high performance aircraft, helicopters, ballistic missiles, and robots. Success in this development facilitates the design of complex Air Force systems.

On the theoretical level, resolution of these questions has involved the development of new mathematical algorithms utilizing transfer matrix factorizations and the associated manipulation of polynomial matrices to reduce the computational requirements necessary to implement advanced control system designs. On the practical level, the revolution in computer hardware has created an exceptional opportunity for the implementation of complex designs, especially through the use of microprocessors.

Research Accomplishments:

Since this research was initiated twelve months ago, significant new insight has been obtained regarding the digital control of multivariable systems. In particular, in [1], the question of deadbeat error control in a multi-input/output configuration was completely resolved for the first time. More specifically, a necessary and sufficient condition was outlined which enables the control designer to construct a feedforward controller which zeros the error between the system output and

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The question of deadbeat error control in a multi-input/output configuration was completely resolved for the first time. More specifically, a necessary and sufficient condition was outlined which enables the control designer to construct a feedforward controller which zeros the error between the system output and reference input in a finite number of discrete steps. A constructive algorithm was outlined for generically achieving a zeroing output feedback control law which requires no dynamic storage registers.  (CONTINUED)		

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ITEM #20, CONTINUED:

A new methodology was developed for adaptively assigning the closed loop poles of both discrete and continuous time linear multivariable systems. The procedure is direct in that a parameterized model of the unknown plant need not be explicitly identified. A new theoretical relationship between the two polynomial matrices which characterize a transfer matrix factorization was developed and shown to resolve a variety of questions involving polynomial matrices in a direct and efficient manner.

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reference input in a finite number of discrete steps. Such a design was shown to be simplified by the initial modelling of overall system and controller dynamical behavior using the inverse z-operator,  $d = z^{-1}$ , as in [2]. The employment of  $d$ , rather than  $z$ , for discrete time control system design appears to offer certain computational advantages in other applications as well and, for this reason, a recent paper [3] addressing many of the intricacies associated with such designs was prepared. It is hoped that these two papers will form a foundation on which new techniques for microprocessor control can be developed.

Some exciting new preliminary investigations could prove to be of significant practical importance in this regard. In particular, the design of conventional controllers for linear discrete time systems implies the implementation of dynamical systems described by difference equations which process sensor data at prescribed sampling rates in order to produce discrete control signals at the same rates. As it turns out, a considerable computational saving can be realized if a "block" of sensor data is processed simultaneously. Such a procedure, known as "block processing," has been used successfully to improve computational efficiency in the signal processing field [4][5] but has yet to be fully exploited with respect to discrete control system design.[6] To illustrate how block processing could be used to reduce shift register requirements in a microprocessor controller, consider an  $n$ -th order linear time-invariant discrete time system described by the "standard" state-space difference equations:  $x(k+1) = Ax(k) + Bu(k)$ ;  $y(k) = Cx(k)$ . If the system

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is both controllable and observable, a discrete time observer could be employed to realize the equivalent of a linear state variable feedback design for (say) arbitrarily positioning all  $n$  closed loop poles of the system. Such a design would involve the implementation of the observer via one or more difference equations; i.e. a discrete dynamical systems which would have to be programmed and actuated on the microprocessor controller. A significant amount of code is required to implement even such a rudimentary control system.[7] Through the use of block processing, however, a significant computational saving could be realized.

In particular, suppose  $u(k) = f_0 y(k)$ ,  $u(k+1) = f_1 y(k+1)$ , ...,  $u(k+q) = f_q y(k+q)$  for each successive  $q$ -dimensional block of output data. Such a control law would require no dynamic storage registers. The important questions to then resolve would be the appropriate choices for  $q$  and the  $f_i$  to achieve some desired closed loop performance. Some important new results have been obtained relative to this question when the requirement is that  $y(k+q+m) = 0$  for all  $m \geq 0$  irrespective of the initial system states. More specifically, in [8] a constructive algorithm is given for generically achieving such a design for  $q = n(n+1)/2$ . It is important to note that the question of achieving specific design goals in the blocked multivariable rather than the blocked scalar case represents a far more challenging one. The potential payoff could be quite significant, however, in that block processing could offer very significant computational benefits when compared to more conventional difference equation designs in

a variety of multivariable control applications, including deadbeat, optimal, adaptive and decentralized designs.

To elaborate, a potentially important application of block processing is in decentralized control. In particular, the control of large flexible structures characterized by numerous bending modes is a most important but difficult task. Such systems are ideally suited to control by numerous localized controllers; i.e. decentralized control. It is well known that decentralized control configurations may produce highly oscillatory or even unstable fixed modes which cannot be affected by conventional, time invariant controllers. [9][10] However, such modes can be controlled in certain cases via time varying controllers. [11] An interesting aspect of block processing, relative to this observation, is that it usually implies a periodic, time-varying system. The results obtained under this grant could have potential impact regarding the design of appropriate time varying controllers.

The collaborative work between Professors Wolovich and Elliott in the area of multivariable adaptive control continued to be productive. In their recent publication [12][13][14], Professors Wolovich and Elliott present a new methodology for adaptively assigning the closed loop poles of both discrete and continuous time linear multivariable systems. The procedure is direct in that a parameterized model of the unknown plant need not be explicitly identified. Rather, the control parameters and an "equivalent" plant parameterization are simultaneously estimated from input-output data using linear parameter estimation procedures. Implementation of the actual design then requires



only knowledge of the controllability indices of the system and an upper bound on the observability index. One of the major practical drawbacks of their work thus far has been the rather extensive computational requirements associated with the actual implementations of their designs. Here again, block processing could prove to be an effective means of reducing these computational requirements in order to employ low cost microprocessors as the primary adaptative control element.

Virtually all of the computational procedures outlined earlier with respect to block processing and adaptive control are based on sound theoretical results, many of these involving transfer matrix factorizations and the associated manipulation of polynomial matrices in either the Laplace operator  $s$ , the delay operator  $d$ , or  $z = d^{-1}$ . In particular, the adaptive control algorithms developed in [12]-[14] rely heavily on theoretical findings involving polynomial matrix state observers, and the actual implementation of these control algorithms requires the "on line" solution of appropriate polynomial matrix equations which converge to unique, stabilizing values.

One way of reducing computational requirements is through the development of more efficient control algorithms. In this regard, some rather illuminating new theoretical results have been recently obtained which appear to have the potential of substantially reducing the computational requirements associated with solving certain polynomial matrix equations.

More specifically, consider a linear multivariable system described by its pxm rational transfer matrix,  $T(s) = R(s)P(s)^{-1}$ ,

where  $R(s)$  and  $P(s)$  are polynomial matrices in the Laplace operator  $s$ , and  $P(s)$  is column proper. Let  $\mu_1, \mu_2, \dots, \mu_m$  represent the controllability indices and  $\nu_1, \nu_2, \dots, \nu_p$  the observability indices associated with this system. Further define the  $n \times m$

polynomial matrix  $(n \triangleq \sum_{i=1}^m \mu_i) S^\mu(s) \triangleq$  block diagonal  $\begin{bmatrix} 1 \\ s \\ \vdots \\ s^{\mu_i-1} \end{bmatrix}$   
 and the  $\bar{n} \times p$  polynomial matrix  $(\bar{n} \triangleq \sum_{j=1}^p \nu_j) S^\nu(s) =$  block diagonal  $\begin{bmatrix} 1 \\ s \\ \vdots \\ s^{\nu_j-1} \end{bmatrix}$

In view of these definitions it can be shown [17] that the following important relationship holds:

$$S^\nu(s) R(s) = \bar{M} S^\mu(s) + \hat{M}(s) P(s), \quad (*)$$

where  $\bar{M}$  is an appropriately "row ordered ( $\bar{n} \times n$ ) observability matrix" of the system and  $\hat{M}(s)$  is an associated polynomial matrix. In particular, given  $R(s)$ ,  $P(s)$ , and the controllability and observability indices, (\*) can readily be solved for  $\bar{M}$  and  $\hat{M}(s)$  using the polynomial matrix division algorithm, as formally shown in [15]. Preliminary investigations indicate that (\*) represents a useful new relationship between  $R(s)$  and  $P(s)$  which can be employed to resolve a variety of questions involving polynomial matrices in a direct and efficient manner.

To illustrate this observation, consider the well known Diophantine equation:

$$H(s)R(s) + K(s)P(s) = F(s), \quad (**)$$

when  $R(s)$ ,  $P(s)$  and  $F(s)$  are known and  $H(s)$  and  $K(s)$  are unknown

and to be determined. It might be noted that this equation forms the basis of the multivariable adaptive control algorithms presented in [12] - [14]. If  $F(s)$  is divided on the right by  $P(s)$ , the polynomial matrix division algorithm yields the following relation:

$$F(s) = \bar{F}S^u(s) + \hat{F}(s)P(s), \quad (+)$$

for some constant matrix,  $\bar{F}$ , and associated polynomial matrix,  $\hat{F}(s)$ . Next, note that  $\bar{M}$  in (\*) will be square and nonsingular if  $R(s)$  and  $P(s)$  are relatively right prime. Under this assumption, therefore, premultiplication of (\*) by  $\bar{F}\bar{M}^{-1}$  and utilization of (+) directly implies that  $\bar{F}\bar{M}^{-1}S^v(s)R(s) = F(s) - \hat{F}(s)P(s) + \bar{F}\bar{M}^{-1}\hat{M}(s)P(s)$ , or that (\*\*) holds with  $H(s) = \bar{F}\bar{M}^{-1}S^v(s)$  and  $K(s) = \hat{F}(s) - \bar{F}\bar{M}^{-1}\hat{M}(s)$ ; i.e. that (\*) and (+) together solve (\*\*). It can further be shown that  $H(s) = \bar{F}\bar{M}^{-1}S^v(s)$  is unique and, perhaps more important, of "minimum degree", suggesting the additional utility of (\*) in directly obtaining proper stabilizing compensators for "open loop" systems described by  $T(s) = R(s)P^{-1}(s)$ . Recent investigations have verified this observation, and a new report which outlines an appropriate synthesis algorithm is currently in preparation.



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